



DCU-003-2011008

Seat No. _____

B. Sc. (Sem. I) (CBCS) (W.E.F. 2019) Examination

August - 2022

Mathematics : MATH - 01 (A)

(Calculus)

Faculty Code : 003

Subject Code : 20211008

Time : $2\frac{1}{2}$ Hours]

[Total Marks : 70

- Instructions :** (1) This question paper contains 10 questions.
(2) Attempt any five questions.
(3) Figure to the right indicates marks of corresponding question.

- 1 (a) Answer the following questions : 4
- (1) Find the value of c , if $f(x) = x^2 - 2x$, $x \in [-1, 3]$ satisfy Roll's theorem.
- (2) Write the formula of Cauchy's Mean Value theorem.
- (3) Give the reason, the function $f(x) = \tan x$, $x \in \left[0, \frac{\pi}{4}\right]$ is not satisfy Roll's theorem.
- (4) Write the expansion of e^x in powers of x .
- (b) Verify Roll's theorem and find the value of c for the 2
function $f(x) = x^2 - 4x + 3$, $x \in [1, 3]$.
- (c) Verify Lagrange's theorem and find the value of c for 3
the function $f(x) = x^3 + x$, $\forall x \in [0, 1]$.
- (d) Expand $\log x$ in ascending powers of $(x-1)$. 5

2 (a) Answer the following questions : **4**

- (1) State Lagrange's Mean Value theorem.
- (2) What is the 3rd term in the expansion of $\sin x$ in powers of x ?
- (3) Whether Roll's theorem is applicable for the function $f(x) = x, x \in [1, 2]$.
- (4) Check the function $f(x) = 6(x-2)^2, x < 2$ is decreasing / increasing?

(b) Verify Cauchy's theorem and find the value of c for **2**

the functions $f(x) = x^3, g(x) = x^2, \forall x \in [1, 2]$.

(c) State and prove Cauchy's Mean Value theorem. **3**

(d) If $f(x) = e^x$ then prove that $0 < \frac{1}{x} \log \frac{e^x - 1}{x} < 1, x > 0$. **5**

3 (a) Answer the following questions : **4**

(1) $\lim_{x \rightarrow 1} \left(\frac{1}{\log x} - \frac{1}{x-1} \right)$ is of the form _____.

(2) The integrating factor of $(1+x^2) \frac{dy}{dx} + 2xy = 4x^2$ is _____.

(3) The order of differential equation

$$\frac{d^2 y}{dx^2} = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}.$$

(4) Define homogeneous differential equation.

- (b) Find $\lim_{x \rightarrow 0} \frac{3^x - 4^x}{x}$. 2
- (c) State and prove Linear Differential equation. 3
- (d) Evaluate : $\lim_{x \rightarrow 0} \frac{\tan x - x}{\tan x - x \sec x}$. 5
- 4 (a) Answer the following questions : 4
- (1) $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{1/x}$ is of the form _____.
- (2) Find $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 - 4}$.
- (3) The integrating factor of $\frac{dx}{dy} = \frac{x}{y+1} + 1$ is _____.
- (4) Define Linear Differential Equation.
- (b) Evaluate : $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$. 2
- (c) Solve : $\frac{dy}{dx} + y \tan x = \sec x$. 3
- (d) Find : $\lim_{x \rightarrow 1} \frac{x^2 - x}{x - 1 + \log x}$. 5
- 5 (a) Answer the following questions : 4
- (1) The general solution of $y(p+1) = xp(p+1) - p^2$ is _____.
- (2) Solve : $p^2 - p - 12 = 0$.
- (3) What is necessary condition for the equation $Mdx + Ndy = 0$ to be exact?
- (4) The general form of Lagrange's differential equation is _____.

- (b) Find the integrating factor of $x^2 \frac{dy}{dx} = 3x^2 - 2xy + 1$. **2**
- (c) Solve : $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$. **3**
- (d) State Bernoulli's equation and derive the method of solution. **5**
- 6** (a) Answer the following questions : **4**
- (1) Whether the differential equation
- $$(3x^2 - 6xy + 2y^2)dx - (3x^2 - 4xy + 6y^2)dy = 0$$
- is exact or not? Verify.
- (2) Solve : $p^2 - 8p + 15 = 0$, where $p = \frac{dy}{dx}$.
- (3) Integrating factor of Bernoulli's equation
- $$x \frac{dy}{dx} + y = x^3 y^6$$
- is _____.
- (4) Solution of $y = px - p^2 + \log p$, where $p = \frac{dy}{dx}$.
- (b) Solve : $\frac{dy}{dx} = xy + x^3 y^3$. **2**
- (c) Solve : $(x^2 - 2xy - y^2)dx - (x + y)^2 dy = 0$. **3**
- (d) Solve : $p^2 - (x + 3y)p + 2y(x + y) = 0$. **5**

7 (a) Answer the following questions : 4

(1) Define Linear Differential Equation of n^{th} order.

(2) If $f(D)y=0$ has complex roots $\alpha \pm i\beta$ then C.F.
= _____.

(3) Evaluate : $\frac{1}{D-2}e^{5x+2}$.

(4) Find the C.F. of $(D^2-16)^2 y = \sin x$.

(b) Solve : $(D^3-9D^2+27D-27)y=0$ _____ **2**

(c) Solve : $(D^2+2D+1)y=2x+x^2$. **3**

(d) Prove in usual notation **5**

$$\frac{1}{f(D)}e^{ax}F(x)=e^{ax}\frac{1}{f(D+a)}F(x).$$

8 (a) Answer the following questions : 4

(1) $\frac{1}{D^2+9}\sin 3x =$ _____.

(2) If the roots of $f(D)y=0$ are 1,1,2,2 then
 $y =$ _____.

(3) $(1-D)^{-2} =$ _____.

(4) $\frac{1}{f(D^2)}\sin ax =$ _____ where $f(-a^2) \neq 0$.

(b) Solve : $(D^2-3D+2)y=e^{2x}$. **2**

(c) Solve : $(D^3+1)y=3+e^{-x}+5e^{2x}$. **3**

(d) Solve : $(D^2-2D+4)y=e^x \cos x$. **5**

9 (a) Answer the following questions : **4**

(1) If m_1, m_2, m_3, m_4 are real roots of $f(D)y = 0$ where

$m_1 = m_2, m_3 \neq m_4$ then C.F. = _____.

(2) $\frac{1}{D^2 + 7} \sin 2x =$ _____.

(3) If $(x^2 D^2 + xD - 9)y = 0$ then C.F. = _____.

(4) $(1 - \theta)^{-2} =$ _____.

(b) Solve : $\frac{d^2 y}{dx^2} = \frac{6y}{x^2}$. **2**

(c) Solve : $(x^2 D^2 - xD + 1)y = \log x$. **3**

(d) Solve : $(x^2 D^2 - xD + 2)y = x \log x$. **5**

10 (a) Answer the following questions : **4**

(1) Integrating factor of $\frac{dy}{dx} + Py = Qy^n$.

(2) Find C.F. of $(x^2 D^2 + xD - 1)y = 0$.

(3) $\frac{1}{D+1} 2x =$ _____.

(4) P.I. of $(x^2 D^2 + 5xD + 4)y = x^4$.

(b) $\frac{1}{D^2-9}e^{3x} = \underline{\hspace{2cm}}.$ **2**

(c) Solve : $(x^2 D^2 - xD + 2)y = x \log x$. **3**

(d) Solve : $(x^3 D^2 + x^2 D)y = x^6 + x^4 + 2x$. **5**
